Notes by :- @jpwebdevelopers

Floyd- Warshall Algorithm

All - Pair Shortest Path

- The all pair shortest path algorithm is also known as Floyd- Warshall Algorithm is used to find all pair shortest path Problem from a given weighted graph.
- This Algorithm follows the dynamic Programming approach to find the shortest path.
- It will generate a matrix, which will represent the minimum distance from any node to all other nodes.
- This Algorithm is works for both the directed and Undirected weighted graphs.

Steps [Algorithm]

(i) Create a matrix where each cell represents the shortest distance between two points.
→ Initially, set the diagonal cells to 0, and all other cells to the distance if there is a direct connection, or infinity if there isn't.

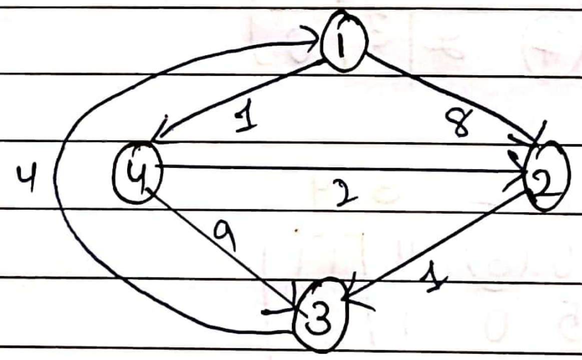
(ii) for each cell in matrix, consider if there's a shorter path from one point to another by passing through a third point.

(iii) If you find a shorter path through a third point, update the cell with this shorter distance.

(IV) Keep checking and updating until you've considered all third points.

(V) The matrix now contains the shortest distance between all pairs of points.

Example



Intial distance matrix :-

$$D_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & \infty & 1 \\ 2 & \infty & 0 & 1 & \infty \\ 3 & 4 & \infty & 0 & \infty \\ 4 & \infty & 2 & 9 & 0 \end{array}$$

$$D_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & \infty & 1 \\ 2 & \infty & 0 & 1 & \infty \\ 3 & 4 & 12 & 0 & 5 \\ 4 & \infty & 2 & 9 & 0 \end{array}$$

Search :- JPNotes

$$D_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & 9 & 1 \\ 2 & \infty & 0 & 1 & \infty \\ 3 & 4 & 12 & 0 & 5 \\ 4 & \infty & 2 & 3 & 0 \end{array}$$

Notes by :- @jpwebdevelopers

$$D_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & 9 & 1 \\ 2 & 5 & 0 & 1 & 6 \\ 3 & 4 & 12 & 0 & 5 \\ 4 & 7 & 2 & 3 & 0 \end{array}$$

Instagram :- @jpwebdevelopers

$$D_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 4 & 1 \\ 2 & 5 & 0 & 1 & 6 \\ 3 & 4 & 7 & 0 & 5 \\ 4 & 7 & 2 & 3 & 0 \end{array}$$

Complexity

Time Complexity :- $O(n^3)$

Space Complexity :- $O(n^2)$

Applications

- To find the shortest path in a directed graph.
- To find the transitive closure of directed graph.

Notes by :- @jpwebdevelopers